The moment problem in here asks if there exists a function $f$ in the Schwarz space $S(\mathbb{R})$ of rapidly decreasing smooth complex functions with support in $[0, \infty)$ (denoted by $S(0, \infty)$) and with arbitrary prescribed moments $\{\mu_n\}_{n=0}^{\infty}$, i.e., such that $\int_0^\infty x^n f(x) \, dx = \mu_n$, $n \in \mathbb{N}_0 = \{0, 1, 2, \cdots\}$. To this question one of previous works gave the following answer: For every $\{\mu_n\}_{n=0}^{\infty} \in \Lambda_\alpha$, there exists $f \in S_\alpha(0, \infty)$ such that $\int_0^\infty x^n f(x) \, dx = \mu_n$, $n \in \mathbb{N}_0$, if $\alpha > 2$.

Here $S_\alpha(0, \infty)$ are classical Gelfand-Shilov spaces, with $\alpha > 0$ consisting of the functions $f \in S(0, \infty)$ such that $\sup_{x \in \mathbb{R}} |x^m f^{(n)}(x)| \leq C_m q^n n!^{\alpha}$, $m, n \in \mathbb{N}_0$ for some $q > 0$ and $C_m > 0$, and $\Lambda_\alpha$ is a Gevrey sequence $\{a_n, n \in \mathbb{N}_0\}$, i.e. such that $|a_n| \leq C_0 q^n n!^{\alpha}$, $n \in \mathbb{N}_0$ for some $q_0 > 0$ and $C > 0$. In this paper the result is extended to the case of general Gelfand-Shilov spaces, $S_M(0, \infty)$ determined by a positive sequence $M = \{M_n; n \in \mathbb{N}_0\}$ which plays the role of $\{n!^{\alpha-1}; n \in \mathbb{N}_0\}$ in the bound of the above inequality used for defining $S_\alpha(0, \infty)$. Also another classical Gelfand-Shilov spaces $S^0(0, \infty)$ are likewise defined to be a space of functions $f \in S(0, \infty)$ such that $\sup_{x \in \mathbb{R}} |x^m f^{(n)}(x)| \leq C_m q^n n!^{\alpha}$. Correspondingly generalized counterparts $S^M(0, \infty)$ are also defined in the similar way. In more detail, we define general Gelfand-Shilov spaces $S_M(0, \infty)$ and $S^M(0, \infty)$ by the formula $S_M = \bigcup_{q \in \mathbb{N}} S_{M,q}$, where $S_{M,q} = \left\{ f \in S(\mathbb{R}) : \sup_{x \in \mathbb{R}, m \in \mathbb{N}_0} \frac{|x^m f^{(n)}(x)|}{q^m n! M_m} < \infty, n \in \mathbb{N}_0 \right\}$ and the formula $S^M = \bigcup_{q \in \mathbb{N}} S^{M,q}$, where $S^{M,q} = \left\{ f \in S(\mathbb{R}) : \sup_{x \in \mathbb{R}, n \in \mathbb{N}_0} \frac{|x^m f^{(n)}(x)|}{q^n n! M_n} < \infty, m \in \mathbb{N}_0 \right\}$. It
can be seen that \( S_{M,q} \) and \( S^{M,q} \) are Frechet spaces with the family of seminorms 
\[
p_n(f) = \sup_{x \in \mathbb{R}, m \in \mathbb{N}_0} \frac{|x^m f^{(n)}(x)|}{q^{n_m M_{n_m}}} \quad \text{for } f \in S_{M,q} \quad \text{and} \quad p^n(f) = \sup_{x \in \mathbb{R}, n \in \mathbb{N}_0} \frac{|x^n f^{(n)}(x)|}{q^n M_{n}}
\]
for \( f \in S^{M,q} \) respectively. It is proved that if \( M \) is increasing sequence of positive numbers, then \( S(S^{M,q}) \subset S_{M,q}, S^{-1}(S^{M,q}) \subset S_{M,q} \) and the map \( S, S^{-1} : S^{M,q} \rightarrow S_{M,q} \) are linear and continuous where \( S \) is the Fourier transform and \( S^{-1} \) its inverse. The first main result in this paper is the following: Suppose that \( M = \{M_n; n \in \mathbb{N}_0\} \) is a sequence of positive real numbers which satisfies (1) non-quasianalyticity condition, i.e. \( \sum_{n=0}^{\infty} \frac{M_n}{(n+1)M_{n+1}} < \infty \), (2) \{n!M_n\}_{n \in \mathbb{N}_0} \) satisfies logarithmically convex condition ( \( M_n^2 \leq M_{n-1}M_{n+1} \)), and \( \{\alpha_n\}_{n \in \mathbb{N}_0} \) is a sequence of distinct positive numbers such that \( \alpha_n \rightarrow \infty \). Then under these assumptions we find that for any complex numbers \( \{\mu_n\}_{n=0}^{\infty} \) there exists \( f \in S^M(0, \infty) \) such that \( \int_0^\infty x^{\alpha_n} f(x)dx = \mu_n \). This result is a generalization of the previous work in two respects: the exponents of \( t \) in the integral defining the moments may be a sequence more general than just \( \{n\}_{n \in \mathbb{N}_0} \) and the function \( f \) may be chosen in the spaces \( S^M(0, \infty) \). The second main result of this paper concerns the general Gelfand-Shilov spaces \( S_{M,q}(0, \infty) \) consisting of functions \( f \in S_{M,q} \) with support in \([0, \infty)\). We should define a sequence \( \Lambda_{M,q} \) instead of Gevery sequence such that \( \Lambda_{M,q} := \left\{ a_n; \sup_{n \in \mathbb{N}_0} \frac{|a_n|}{q^n M_n} < \infty \right\} \). It is shown that \( M \) is a sequence of moderate growth, then there exists \( c \in \mathbb{N} = \{1, 2, 3, \cdots\}, c > 1 \) such that whenever \( f \in S_{M,q} \) \( (0, \infty) \) for some \( q \in \mathbb{N} \), one has \( \mathcal{M}(f) = \{\mu_n(f)\}_{n \in \mathbb{N}_0} \in \Lambda_{M,q} \) and the moment map \( \mathcal{M} : S_{M,q} \) \( (0, \infty) \rightarrow \Lambda_{M,q} \) is continuous. Here moderate growth of \( M \) means that there exists \( A > 0 \) such that \( M_n \leq A^n \inf_{k,l \in \mathbb{N}_0, k+l=n} M_k M_l, n \in \mathbb{N}_0 \). The last statement has the converse:

If the sequence \( M \) has the properties (3) increasing, (4) logarithmically convex, (5) of moderated growth, (6) strong non quasi-analiticity (i.e. there exists \( B > 0 \) such that \( \sum_{n \geq n_0} M_n / (n+1)M_{n+1} \leq B M_n M_{n+1}, n \in \mathbb{N}_0 \)) and (7) the growth index \( \gamma(M) > 1 \), then there exists \( c \in \mathbb{N} \), for every \( q \in \mathbb{N} \) there is a continuous linear map \( T_{M,q} : \Lambda_{M,q} \rightarrow \Lambda_{M,q} \) \( (0, \infty) \) and \( (M \circ T_{M,q}) = I_{M,q} \). Idea of the proof is the following: Given \( \mu = \{\mu_n\}_{n \in \mathbb{N}_0} \in \Lambda_{M,q} \), we are searching for a function \( T_{M,q}(\mu) \in S_{M,q}(0, \infty) \) such that \( \int_0^\infty t^n T_{M,q}(\mu)(t)dt = \mu_n, n \in \mathbb{N}_0 \). Hence if \( \psi = \mathcal{F}(T_{M,q}(\mu)) \), it satisfies \( \psi^{(n)}(0) = (-i)^n \int_0^\infty t^n T_{M,q}(\mu)(t)dt = (-i)^n \mu_n \).

Hence we only need to construct \( \psi(t) \) and put \( T_{M,q}(\mu) = \mathcal{F}^{-1}(\psi) \). To find out the \( \psi \), we use results, proved in this paper, on the existence of continuous right inverse for the Borel map which is obtained for ultraholomorphic classes.
in sectors. Finally, we can say if $M$ satisfies all condition (3)-(7), then the map $\mathcal{M} : S_M (0, \infty) \to \Lambda_M$ is onto and every Stieltjes moment problem has a solution in general Gelfand-Shilov spaces. The authors comments that the condition $\gamma (M) > 1$ is also necessary to get the main result.