This paper aims to extend the method of Ran and Reurings (A nonlinear matrix equation connected with interpolation theory, Linear Alg. Appl, 2004) to matrix extremal interpolation problems, especially Schur problem, Nevanlinna-Pick problem and Jordan blocks. Schur extremal problem: Let the complex matrices \( a_0, a_1, \ldots, a_p \) be given. We seek an \( m \times m \) matrix valued function \( w(z) \), holomorphic in \(|z| < 1\), satisfying
\[
\begin{align*}
  w(z) &= a_0 + a_1 z + \cdots + a_p z^p + \cdots \\
  w^*(z) w(z) &\leq \rho_{\min}^2,
\end{align*}
\]
where \( \rho_{\min} \) is a positive Hermitian matrix defined by the minimum rank condition. For
\[
S_1 = C_p^* C_p, \quad C_p = \begin{pmatrix}
  a_0 & a_1 & \cdots & a_p \\
  0 & a_0 & \cdots & a_{p-1} \\
  \vdots & \ddots & \ddots & \ddots \\
  0 & \cdots & 0 & a_0
\end{pmatrix}
\]
and
\[
S_2 = I_{(p+1)m}, \quad R = \text{diag}(\rho, \cdots, \rho)
\]
is said to satisfy the minimum rank condition if \( RS_2 R - S_1 \succeq 0 \) and it minimizes the rank of \( RS_2 R - S_1 \) and written as \( R_{\min} = \text{diag}(\rho_{\min}, \cdots, \rho_{\min}) \). Nevanlinna-Pick extremal problem: Let the complex matrices \( \eta_1, \eta_2, \ldots, \eta_n \), the points \( z_1, z_2, \ldots, z_n \) satisfying \(|z_k| < 1, k = 1, 2, \ldots, n\). We seek an \( m \times m \) matrix valued function \( w(z) \), holomorphic in \(|z| < 1\), such that
\[
\begin{align*}
  w(-z_k) &= \eta_k^*, k = 1, 2, \ldots, n \\
  w^*(z) w(z) &\leq \rho_{\min}^2,
\end{align*}
\]
where \( \rho_{\min} \) is defined by the minimum rank condition. This time, for
\[
S_1 = \left( \frac{\eta_1 \eta_1^*}{1-z_k \bar{z}_l} \right)_{k,l=1}^n
\]
and
\[
S_2 = \left( \frac{I_m}{1-z_k \bar{z}_l} \right)_{k,l=1}^n, \quad R_{\min} = \text{diag}(\rho_{\min}, \cdots, \rho_{\min})
\]
minimizes the rank of \( RS_2 R - S_1 \). These extremal problem can be solved by using some Riccati type equations for the matrix \( \rho_{\min} \) and be assured to have one and only one solution. Also Riccati type equation is treated for Jordan blocks. The actual procedure to compute \( \rho_{\min} \) is being shown.