A parametrized family of solutions of the finite moment problem for rational matrix-valued functions is provided and its extremal property is discussed in several directions. The solutions constitute non-negative Hermitian matrices belonging to $M_{q \times q}^\geq (T, B_T)$, the set of all nonnegative $q \times q$ matrix measures on the $\sigma$-algebra $B_T$ of all Borel subsets of the unit circle $T$. Let $n$ be a positive integer. Let $(\alpha_j)_{j=1}^n$ be a sequence of complex numbers belonging to $C \setminus T$ and let $\pi_{\alpha,n}(u) = \prod_{j=1}^n (1 - \bar{\alpha}_j u)$. Denote $R_{\alpha,n}$ the set of all rational functions which admit a representation $f = P \pi_{\alpha,n}$ with some polynomial $P$ of degree not greater than $n$ and $R_{q \times q}^{m \times n}$ the set of all $q \times q$ matrix whose entries belong to $R_{\alpha,n}$. If $(X_k)_{k=0}^n$ is a sequence of matrix valued functions which belong to $C_{q \times q}$-module $R_{q \times q}^{m \times n}$, we would construct $G_{X,n}^{(F)} = \left( \int_T (X_j(z))^s F(\frac{dz}{2\pi}) X_k(z) \right)_{j,k=0}^n$. Our problem is stated in the following way: For a given $(n+1)q \times (n+1)q$ complex matrix $G$, describe the set $M \left[ (\alpha_j)_{j=1}^n, G; (X_k)_{k=0}^n \right]$ of all measures $F \in M_{q \times q}^\geq (T, B_T)$ such that $G_{X,n}^{(F)} = G$. For the ordinary moment problem, we could take the basis $X_k = I_q$, $k = 0, 1, 2, \cdots, n$ where $I_q$ is identity $q \times q$ matrix. For each $w \in D \setminus \bigcup_{j=1}^n \left\{ \frac{1}{\alpha_j} \right\}$, our aimed extremal solution is seen to be $F_{\alpha,n}^{(n)}(B) = \frac{1}{2\pi} \int_B \frac{1-|w|^2}{|z-w|^2} A^{n-1}(z) A(w) A^{-1}(z) \lambda(\frac{dz}{2\pi})$ for $B \in B_T$, where $\lambda$ is the normalized Lebesgue measure on $T$ and $A(v) = \Xi_n(v) G^{-1} \Xi_n^\dagger(w)$ is being defined with $\Xi_n = (X_0, X_1, \cdots, X_n)$. The extreme property can be explained by using a maximal entropy. If $\Lambda(z) \frac{\lambda(\frac{dz}{2\pi})}{2\pi}$ is the absolutely continuous part of the Lebesgue de-
composition for \( F \in M^q_\mathbb{Z} (T, \mathcal{B}_T) \), we define the entropy of \( F \) with respect to the point \( w \) as 
\[ \tilde{\eta}_w (F) = - \frac{1}{4 \pi} \int_T \frac{1 - |w|^2}{|z-w|^2} \log (\det \Lambda (z)) \lambda (dz). \]
Then we can prove an inequality 
\[ \tilde{\eta}_w (F) \geq - \frac{1}{2} \log \left( \det \left( h_n^{(\alpha)} (w) \right) \left( 1 - |w|^2 \right) A^{-1} (w) \right) \lambda (dz), \]
where 
\[ h_n^{(\alpha)} (w) = \prod_{j=1}^n \frac{\alpha_j - w}{1 - \alpha_j w}, \]
and equality holds if and only if \( F = F^{(\alpha)}_{n,w} \). In other words the solution \( F = F^{(\alpha)}_{n,w} \) gives a maximal entropy for each parameter \( w \in D \). The extremal properties concern not only the entropy but also some maximality property of right and left outer spectral factors with respect to the Lowner semiordering. It is also shown that the values of the reproducing kernels associated with \( F = F^{(\alpha)}_{n,w} \) have an extremal property with respect to the Lowner semiordering.